Theory session - 2

Noise suppression
<table>
<thead>
<tr>
<th>E-BEAM NOISE AND RADIATION SUPPRESSION THEORY</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Microwave tube noise suppression</strong></td>
</tr>
<tr>
<td><strong>Optical noise suppression in a drifting relativistic beam:</strong></td>
</tr>
<tr>
<td>Gover, Phys. Rev. Lett. 102, 154801 (2009),</td>
</tr>
<tr>
<td>Nause, JAP, 107, 103101 (2010)</td>
</tr>
<tr>
<td><strong>Optical noise suppression with a dispersive section:</strong></td>
</tr>
<tr>
<td>Rathner, PhysRevSTAB 14 060710 (2011)</td>
</tr>
<tr>
<td>Gover, Phys. Plasmas 18, 123102 (2011)</td>
</tr>
<tr>
<td><strong>SASE noise suppression:</strong></td>
</tr>
<tr>
<td>Gover, JQE46, 1511 (2010)</td>
</tr>
<tr>
<td><strong>Short wavelength limit:</strong></td>
</tr>
<tr>
<td>R. Bonifacio, Optics Communications 138 (1997) 99-100</td>
</tr>
<tr>
<td>K-J Kim, Shanghai FEL conférence 2011</td>
</tr>
</tbody>
</table>
1D Dispersive Shot Noise Suppression

\[ N \langle |b(k)|^2 \rangle = (1 - \gamma)^2 \]

\[ \gamma \equiv n_0 R_{56} A \]

OTR proportional to charge at start

QB at 10.3 kG

Simulated OTR

Measured OTR

Normalized OTR, \( b(k) \)

OTR Data
Linear Fit

Dispersive Shot Noise Supp.
**DIMENSIONLESS SCALING PARAMETERS**

- **Energy spread parameter**
  Longitudinal displacement due to energy spread in a 1-d plasma period / microbunching wavelength.

- **Emittance parameter**
  Longitudinal displacement due to transverse emittance in a 1-D plasma period / microbunching wavelength.

  **Cold beam limit:** $K_\gamma, K_\varepsilon \ll 1$

- **3-D parameter**
  Transverse beam size/ microbunching wavelength in the rest frame.

  **edge effects are negligible if** $D \gg 1$

- **Focusing parameter**
  Betatron frequency/1-D plasma frequency

  **Transverse motion negligible if:** $K_\beta \ll 1$

  (laminar beam limit)
SUMMARY AND CONCLUSIONS

- We have developed a six-dimensional theory of the space-charge eigenmodes of a beam with finite emittance, betatron motion, energy-spread and 3-D field effects.
  - new results:
    - emittance induced Landau damping;
    - coupling of space-charge to betatron motion.

- The theory of space-charge waves has been used to develop a fully kinetic model of the space-charge induced microbunching instability.
  - Self-consistent analysis of SC amplification
  - emittance effects in space-charge amplification/suppression of microbunching;

- A microbunching reconstruction experiment, based on phase-retrieval techniques, has been designed and performed at the NLCTA test facility at SLAC.
  - successful reconstruction of seeded microbunching with application to compressed beams
**KJK Talk Summary (Micro-bunching WS)**

- SASE including quantum effect:
  \[
  \frac{dP}{d\omega} = \frac{\rho E e}{2\pi} G\left(\chi_C + q\langle F_Q^+ F_Q \rangle\right)
  \]

- \[
  \chi_C(z_0) = \frac{1}{N_e} \left| \sum_j e^{-ikz_j(z_0)} \right|^2
  \]
  This is the full initial noise term in SASE.
  In general it *cannot* be separated into the position noise and the momentum noise.

- If \( \eta_j / \mu \rho \ll 1 \) then the decomposition is feasible:
  \[
  \chi_C = \chi_{C\zeta} + \chi_{C\eta}
  \]
  \[
  \chi_{C\zeta}(z_0) = N_e \left| \sum_j e^{-ikz_j(z)} \right|^2,
  \quad \chi_{C\eta}(z_0) = N_e \left| \sum_j \Delta\beta_j e^{-ikz_j(z)} \right|^2
  \]


**Classical SASE noise**

- \( b_k(z) = \frac{1}{N_e} \sum_j e^{-i k \zeta_j(z)} \) was calculated in terms of initial conditions by solving Klimontovich-Gauss equations. There are two different parts: \( b_k = b_{k,PO} + b_{k,RIM} \)

- Collective motion giving rise to plasma oscillation

\[
\begin{align*}
    b_{k,PO}(z) &\approx b_k(0) \cos(\Omega_P z) - i \frac{k}{\Omega_P} p_k(0) \sin(\Omega_P z) \\
    p_{k,PO}(z) &\approx -i \frac{\Omega_P}{k} b_k(0) \sin(\Omega_P z) + p_k(0) \cos(\Omega_P z)
\end{align*}
\]

- Random individual motion (RIM): \( b_{k,RIM}(z) = \frac{1}{N_e} \sum_i \frac{e^{-i(k\zeta_0^i + \Delta \beta^0_0 i z)}}{\varepsilon(k,k\Delta \beta_0^0 i)} \)

where \( \varepsilon \) is the plasma dielectric function

- The RIM part of the noise can be calculated analytically, assuming the momentum distribution is Gaussian:

\[
\langle |b_k^I|^2 \rangle = \frac{1}{N_e} \int d\Delta \beta \frac{g(\Delta \beta)}{|\varepsilon(k,k\Delta \beta)|^2} = \frac{1}{N_e} \frac{(k\lambda_D)^2}{1+(k\lambda_D)^2}, \quad \lambda_D = \sigma_{\Delta \beta}/\Omega_P: \text{Debye length}
\]
Classical SASE Noise

- The RIM part is small if $k\lambda_D$ is small (1% if $k\lambda_D=0.1$).
- Otherwise the RIM part dominates and noise reduction will not work since the RIM part cannot be controlled.
- For the LCLS parameters, assuming the rms energy spread to be 0.01%, $k\lambda_D$ is about 1.
- The angular divergence also gives rise to longitudinal velocity spread, modifying the value of the Debye length. In the case of LCLS, the normalized emittance needs to be smaller than 0.1 mm-mrad for the angular effect to be smaller than the energy spread effect.
- The analysis of the coherent part gives rise to the same results as derived by A. Gover.
Quantum part

- \( q = \frac{\hbar \omega}{\rho E_e} \) gives rise to \( \frac{dp}{d\omega} = \frac{\hbar \omega}{2\pi} \), corresponding to one quantum per input mode as the minimum quantum noise. The one quantum comes from two contributions, \( \frac{1}{2} \) from the vacuum fluctuation and \( \frac{1}{2} \) from the response of the gain medium (electron beam).

- \( q \sim 0.002 \) for LCLS, small but not negligible. Can this quantum effect be measured?

- To obtain the minimum quantum noise, \( \langle F_q^+ F_q \rangle = 1 \), the wave function should be \( \psi(\Delta \theta) = \frac{1}{\sqrt{2\pi q}} \exp[-(\lambda_3^*/q)\Delta \theta^2] \), \( \lambda_3 = (1 + i\sqrt{3})/2 \). Does this corresponds to actual wavefunction from practical electron guns? If it is much different, \( \langle F_q^+ F_q \rangle \gg 1 \), and the quantum effect could be much larger.

classical and quantum KJK Summary
Conditions for **NOISE** suppression


\[ \text{gain} = \frac{|\tilde{i}(L_d, \omega)|^2}{|\tilde{i}(0, \omega)|^2} = \cos^2 \phi_p + N^2 \sin^2 \phi_p \]

\[ \frac{|\tilde{V}(L_d, \omega)|^2}{W_d^2|\tilde{i}(0, \omega)|^2} = \sin^2 \phi_p + N^2 \cos^2 \phi_p \]

\[ N^2 = \frac{|\tilde{V}(0, \omega)|^2}{W_d^2|\tilde{i}(0, \omega)|^2} = \left( \frac{\lambda_D}{\lambda} \right)^2 \]

\[ k_D = \frac{2\pi}{\lambda_D} = \frac{\omega_{pl}}{\delta v} \]

\[ \text{gain}(\phi_p = \pi/2) = N^2 \]

For significant suppression (and negligible Landau damping):

\[ N = \left( \frac{\lambda_D}{\lambda} \right) << 1 \]

Additional conditions:

\[ n_0 A_e \lambda >> 1 \]

Ballistic condition:

\[ \Delta \varphi_p = kL_d \Delta \left( \frac{1}{\beta_z} \right) << 1 \Rightarrow \]

\[ \frac{\Delta \gamma}{\gamma} << \frac{\gamma_0^2 \lambda}{2L_d} \]

\[ \varepsilon_n << \gamma_0 \sigma_0 \left( \frac{\lambda}{L_d} \right)^{\frac{1}{2}} \]
Conditions for **NOISE** suppression

\[ E = 13\text{GeV} \quad I_0 = 3400\text{Amp} \quad \delta E \sim 0.3\text{MeV} \quad \lambda = 1.5\text{A} \]

For significant suppression:

\[ N = \left( \frac{\lambda D}{\lambda} \right) \sim 0.03 \ll 1 \]

Additional conditions:

\[ n_0 A_e \lambda = \frac{I_0}{e c} \lambda \sim 10^4 \gg 1 \]

Ballistic condition:

\[ \Delta \phi_p = k L_d \Delta \left( \frac{1}{\beta z} \right) \ll 1 \Rightarrow \begin{cases} \frac{\Delta \gamma}{\gamma} \ll \frac{\gamma_0^2 \lambda}{2 L_d} \\ \varepsilon_n \ll \gamma_0 \sigma_{x_0} \left( \frac{\lambda}{L_d} \right)^{1/2} \end{cases} \]

SASE suppression factor:

\[ \left( \frac{S}{2} \right)^2 = \left( \frac{\gamma_0}{\gamma z} \frac{\theta_{pw}}{\Gamma} \right)^2 \ll 1 \]
Fundamental “Schawlow-Townes” Coherence Limits (NEP)

e-Beam current noise + energy shot + radiation noise:

$$\left( \frac{dP_{in}}{d\omega} \right)_{\text{min}} = A \cdot e I_b + B \cdot \delta E_c + \frac{\hbar \omega}{1 + e^{-\hbar \omega/kT}}$$

Minimum (energy spread limited) e-beam noise:

$$\left( \frac{dP_{in}}{d\omega} \right)_{\text{min}} = \frac{\delta E_c}{\pi} + \frac{\hbar \omega}{1 + e^{-\hbar \omega/kT}}$$

Microwave/THz regime:

$$\left( \frac{dP_{in}}{d\omega} \right)_{\text{min}} = \frac{\delta E_c}{\pi} + k_B T \quad (\approx \frac{\delta E_c}{\pi} = \frac{k_B T}{\pi} > k_B T)$$

(optical/X-UV regime:

$$\left( \frac{dP_{in}}{d\omega} \right)_{\text{min}} = \frac{\delta E_c}{\pi} + \hbar \omega \quad (\approx \hbar \omega)$$

(Cathode temperature limited)
Coherent Plasma Oscillation in an e-Beam Drift Section

\[
\tilde{i}(L_d, \omega) = \left[ \tilde{i}(0, \omega) \cos \phi_p - i \tilde{V}(0, \omega) \frac{\sin \phi_p}{W_d} \right] e^{i \phi_b(L_d)}
\]

\[
\tilde{V}(L_d, \omega) = \left[ -i i \tilde{i}(0, \omega) W_d \sin \phi_p + \tilde{V}(0, \omega) \cos \phi_p \right] e^{i \phi_b(L_d)}
\]

\[
\tilde{V}(z, \omega) = -\left( mc^2 / e \right) \gamma(z, \omega) = -\left( mc^2 / e \right) \gamma^3 \gamma_0 v_0 \tilde{V}(\omega)
\]

(Chu’s Relativistic Kinetic Voltage)

\[
\phi_b = \frac{\omega}{v_z} L_d \\
\phi_p = \theta_{pr} L_d \\
\theta_{pr} = r_p \frac{0^\prime}{v_0}
\]

\[
\omega_p^\prime = \left( \frac{e^2 n_0}{m \epsilon_0 \gamma^3} \right)^{1/2}
\]

\[
W_d = r_p^2 \sqrt{\mu_0 / \epsilon_0} / k \theta_{prd} A_e
\]
Spatially Coherent interaction condition

Spatially coherent interaction:

\[ \lambda \beta \gamma \lesssim 2r_b \]
Conditions for **SASE** suppression

**IEEE JQE, 46, 1511, 2010**

Drift + Wiggler

\[
\begin{pmatrix}
\tilde{C}_q(z) \\
\tilde{I}(z) \\
\tilde{V}(z)
\end{pmatrix} =
\begin{pmatrix}
H^{EE} & H^{EI} & H^{EV} \\
H^{HE} & H^{H} & H^{HV} \\
H^{VE} & H^{VI} & H^{VV}
\end{pmatrix} \cdot
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \phi_p & \frac{i}{W_d} \sin \phi_p \\
0 & -iW_d \sin \phi_p & \cos \phi_p
\end{pmatrix} \cdot
\begin{pmatrix}
\tilde{C}_q(0) \\
\tilde{I}(0) \\
\tilde{V}(0)
\end{pmatrix}
\]

\[
S = \frac{iW_d H^{EV}}{H^{EI}} = \frac{\gamma_0 \theta_{pw}}{\gamma_z \Gamma}
\]

Minimum SASE radiation power when \( \phi_p = \frac{\pi}{2} - \frac{\sqrt{3}}{2} S \)

Current noise dominance (in SASE): \( N < S < 1 \)

\[
\left( \frac{dP_{in}}{d\omega} \right)^{eff} = \left( \frac{S}{2} \right)^2 \left( \frac{dP_{in}}{d\omega} \right)^I_{conv} = \frac{2 \gamma_0^2}{\pi \gamma_z^2 k \Gamma A_e} \frac{1}{eI_b}
\]

Velocity noise dominance (in SASE): \( S < N < 1 \)

\[
\left( \frac{dP_{in}}{d\omega} \right)^{eff} = \frac{2 \gamma_z^2}{\pi \gamma_0^2 k \Gamma A_e} \frac{(mc^2 \delta \gamma)^2}{eI_b}
\]
NOISE INPUTS INTO FEL AMPLIFIER
(NOISE EQUIVALENT POWER - NEP)

Coherent seed radiation input $\left( P_{in} \right)^{\text{sig}}_q$

Effective e-beam noise
\[
\left( \frac{dP_{in}}{d\omega} \right)^{\text{e-beam}}_q = \left( \frac{dP_{in}}{d\omega} \right)^{\text{SASE}}_q / G
\]

Radiation noise
\[
\left( \frac{dP_{in}}{d\omega} \right)^{\text{QED}}_q = \frac{\hat{n}\omega}{1 + e^{-\omega / kT}}
\]

Partially coherent output radiation
At the Cathode ("virtual" Cathode)

Current and velocity noise – uncorrelated:

\[
|\bar{i}(\omega)|^2 = \frac{1}{T} \left\langle |\bar{i}(\omega)|^2 \right\rangle_{NT} = eI_b
\]

\[
|\bar{v}(\omega)|^2 = \frac{1}{T} \left\langle |\bar{v}(\omega)|^2 \right\rangle_{NT} = \frac{(\delta E_c)^2}{eI_b}
\]

\[
\left(\frac{|\bar{I}(\omega)|^2}{\bar{V}(\omega)|^2}\right)^{1/2} = \delta E_c
\]
A sufficient condition for quarter plasma wavelength oscillation is space-charge dominated transport of a beam through a waist. This is also the condition for maximum noise suppression, if the beam noise is initially uncorrelated and current shot-noise dominated.
BEAM ENVELOPE EXPANSION

\[
\frac{d^2 r_b(z)}{dz^2} - \frac{K}{r_b(z)} - \frac{\varepsilon^2}{r_b^3(z)} = 0
\]

K-V Beam envelope equation (uniform beam)

- \( r_b \) – beam radius
- \( \varepsilon \) – emittance (NON-normalized)
- relativistic perveance

\[
K = \frac{2I_0}{I_A (\gamma \beta)^3}
\]

For Gaussian distributed e-beam:

\[
r_b = \sqrt{2\sigma_x}
\]

\[
\varepsilon = 2\sigma_x \sigma_x'
\]
Waist Definition

Space Charge Dominated Regime ($\varepsilon = 0$)

Numerical solution  Analytic approximation: $r_b(z) = r_0 \left(1 + \frac{K}{2} \left(\frac{z}{r_0}\right)^2\right)$

![Graph showing normalized beam radius and axial distance](image)
DRIFT/DISPERSION TRANSPORT

- Drift
- Dispersive Section

$L_d$ Drift

$L_m$ Dispersive Section

chicane
Control over current noise from suppression to gain with adjustment of drift length and $R_{56}$

\[
K_d = \frac{\left| R_{56} \gamma_0^2 \right|}{L_d}
\]
SUB-RADIANCE
Coherence in Spontaneous Radiation Processes

R. H. Dicke

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received August 25, 1953)

In the usual treatment of spontaneous radiation by a gas, the radiation process is calculated as though the separate molecules radiate independently of each other. To justify this assumption it might be argued that, as a result of the large distance between molecules and subsequent weak interactions, the probability of a given molecule emitting a photon should be independent of the states of other molecules.

This simplified picture overlooks the fact that all the molecules are interacting with a common radiation field and hence cannot be treated as independent. The model is wrong in principle and many of the results obtained from it are incorrect.

\[ \frac{dP_{in}}{d\omega} \propto N \quad \text{Spontaneous emission (radiation noise)} \]

\[ \frac{dP_{in}}{d\omega} \propto \alpha N \quad \rightarrow \quad N^2 \quad \text{Super-radiance (coherent emission)} \]

\[ \frac{dP_{in}}{d\omega} \propto \alpha N \quad \rightarrow \quad 0 \quad \text{Sub-radiance} \]
Fundamental processes of uncorrelated and correlated spontaneous radiative emission in complex $C_q$ plane

- **Spontaneous emission**
- **Superradiance**
- **Subradiance**
SASE SUPPRESSION

Effective Input noise

\[
\left( \frac{dP_{in}^{noise}}{d\omega} \right)_{eff} = \left( \frac{dP(L_w)}{d\omega} \right)_{incoh}/G(\omega)
\]

Coherence condition

\[
[P_s(0)]_{coh} \gg \left( \frac{dP_{in}^{noise}}{d\omega} \right)_{eff} \Delta \omega
\]

To dominate Current Shot-Noise:

\[
[P_s(0)]_{coh} \gg \left( \frac{eI_b Z_0}{16\pi A_{em}} \left( \frac{a_w}{\gamma \beta_z \Gamma} \right)^2 \right) \Delta \omega
\]

\[
|\tilde{i}_s(0)|^2 \gg eI_b \Delta \omega
\]

(Pre-bunching)

(Seed radiation injection)
As for the FEL amplifier operating in the visible or shorter wavelength range, its noise properties are defined only by the shot noise.

Conditions for **SASE** suppression

**IEEE JQE, 46, 1511, 2010**

**Drift + Wiggler**

\[
\begin{pmatrix}
\tilde{C}_q(z) \\
\tilde{I}(z) \\
\tilde{V}(z)
\end{pmatrix} =
\begin{pmatrix}
H^{EE} & H^{EI} & H^{EV} \\
H^{IE} & H^{II} & H^{IV} \\
H^{VE} & H^{VI} & H^{VV}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \phi_p & \frac{i}{W_d} \sin \phi_p \\
0 & -iW_d \sin \phi_p & \cos \phi_p
\end{pmatrix}
\begin{pmatrix}
\tilde{C}_q(0) \\
\tilde{I}(0) \\
\tilde{V}(0)
\end{pmatrix}
\]

\[
S = \frac{iW_d H^{EV}}{H^{EI}} = \frac{W_d \theta_{pw}}{W_w \Gamma} = \frac{\theta_{pw}}{\theta_{pd} \Gamma} = \frac{\gamma_0^{3/2}}{(\gamma_0 \gamma_z^2)^{1/2}} \frac{\theta_{pw}}{\gamma_z \Gamma} = \frac{\gamma_0}{\gamma_z} \frac{\theta_{pw}}{\Gamma}
\]

Minimum SASE radiation power when \( \phi_p = \frac{\pi}{2} - \frac{\sqrt{3}}{2} S \)

**Current noise dominance (in SASE):** \( N < S < 1 \)

\[
\left( \frac{dP_{in}}{d\omega} \right)^{eff} = \left( \frac{S}{2} \right)^2 \left( \frac{dP_{in}}{d\omega} \right)^I_{\text{conv}} = \frac{2}{\pi \Gamma} \frac{W_d^2}{W_w} eI_b = \frac{2}{\pi} \frac{\gamma_0^2}{\gamma_z^2} \frac{1}{k\Gamma A_e} eI_b
\]

**Velocity noise dominance (in SASE):** \( S < N < 1 \)

\[
\left( \frac{dP_{in}}{d\omega} \right)^{eff} = \frac{2}{\pi \theta_{pw}} \frac{\Gamma}{W_d^2} \frac{mc^2 \delta \gamma^2}{eI_b} = \frac{2}{\pi} \frac{\gamma_z^2}{\gamma_0^2} k\Gamma A_e \frac{(mc^2 \delta \gamma)^2}{eI_b}
\]
Conditions for **NOISE** suppression

**LCLS (no heating)**

\[ E = 13 GeV \quad I_0 = 3400 Amp \quad \delta E \sim 0.3 MeV \quad \lambda = 1.5 A \]

For significant suppression:

\[ N = \left( \frac{\lambda_D}{\lambda} \right) \sim 0.03 \ll 1 \]

Additional conditions:

\[ n_0 A_e \lambda = \frac{I_0}{ec} \lambda \sim 10^4 \gg 1 \]

Ballistic condition:

\[ \Delta \varphi_p = k L_d \Delta \left( \frac{1}{\beta_z} \right) \ll 1 \Rightarrow \left\{ \begin{array}{l}
\frac{\Delta \gamma}{\gamma} \ll \frac{\gamma_0^2 \lambda}{2 L_d} \\
\varepsilon_n \ll \gamma_0 \sigma x_0 \left( \frac{\lambda}{L_d} \right)^{1/2}
\end{array} \right. \]

Noise suppression factor:

\[ \left( \frac{S}{2} \right)^2 = \left( \frac{\gamma_0 \theta_{pw}}{\gamma_z \Gamma} \right)^2 \ll 1 \]