Theoretical approaches to microbunching instability

Gennady Stupakov
SLAC National Accelerator Laboratory, Stanford, CA 94309

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Outline of the talk

- Introduction
- Microbunching in rings
- Wakefields causing microbunching
- Microbunching in a bunch compressor
- Correlation functions technique
- Noise suppression—an example of “anti-microbunching”
- Summary
CSR instability in LCLS BC2

Courtesy H. Loos.

µB1 hinders OTR diagnostics

Normal operating condition

OTR11, OTR21 inserted, no coherence


Microbunching in linear accelerators is a relative of the well known *microwave* instability in circular machines. It is called microbunching because the typical wavelength of the modulation is much shorter than the bunch length.

Instability is excited by the wake fields in the machine.
A simple model of microwave instability (in a ring)

- Because the wavelength is much smaller than the bunch length, assume a coasting beam with current $I_0$
- Neglect the energy spread in the beam—a cold beam model
- Take into account the slippage: particles with different energy are moving relative to each other in the longitudinal direction
- Assume a thin beam, beam radius $\rightarrow 0$
- Consider a small sinusoidal perturbation on the beam, $n(z, t) = n_0 + n_1(z, t)$ with $n_1 \ll n_0$ ($n_0 = I_0/ec$, for relativistic beam)
A simple model of microwave instability (in a ring)

- Linearized continuity equation

\[
\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_{sl}}{\partial z} = 0
\]

- The slippage “velocity” \( v_{sl} \) is proportional to \( \delta = \Delta E/E_0 \) and the slippage \( \eta \)

\[
v_{sl}(z, t) = -c\eta\delta(z, t)
\]

- The energy change is due to the wakefield (per unit length of path), \( \delta = \Delta E/E_0 \)

\[
\frac{\partial \Delta E(z, t)}{\partial t} = -ce^2 \int w(z' - z)n_1(z', t)dz'
\]

- Assume all the perturbations \( \propto e^{-i\omega t + ikz} \). Instead of the wake we get the impedance

\[
Z(ck) = \frac{1}{c} \int w(z)e^{ikz}dz
\]
A simple model of microbunching instability (in a ring)

The dispersion relation

\[ \omega^2 = i \frac{c^3 r_e n_0 k \eta}{\gamma} Z(ck) \]

The imaginary part \( \Gamma = \text{Im} \omega \) gives the growth rate of the instability. We see that almost any impedance will cause an instability if \( Z \) is complex (only for purely inductive impedance \( Z = -i\omega L \) the beam is stable). The growth rate scales as \( \Gamma \propto \sqrt{I_0} \).

- Due to the wake \( \Delta I \) induces energy modulation in the beam, \( \Delta E = eZ \times \Delta I \)
- Momentum compaction of the ring translates \( \Delta E \) into \( \Delta n \rightarrow \Delta I \). Under certain conditions, the final \( \Delta I \) is greater than the initial one.
CSR wake—1D steady state

A relativistic particle moving in vacuum in a circular orbit of radius $R$, in steady state, generates a CSR wake (per unit length of path) (Logansen and Rabinovich, 1960; Murphy, Krinsky and Gluckstern, 1995; Derbenev et al., 1995).

The wake is localized in front of the particle, $z > 0$.

For $z \ll R$ and $z \gg R/\gamma^3$,

$$w(z) \approx -\frac{E_\parallel}{e} = -\frac{2}{3^{4/3}R^{2/3}z^{4/3}}$$

Area under $w(z)$ is zero.
Approximations made in 1D steady state CSR wake

Approximations:

- Metallic walls are far enough from the orbit—the shielding effect is neglected.
- Small transverse beam size, $\sigma_\perp \lesssim l_\perp \ll (\lambda^2 R)^{1/3}$ (here $\lambda = \lambda/2\pi$).
- Transient effects at the entrance to and exit from the magnet are neglected:

$$l_{\text{magnet}} > l_f \sim (\lambda R^2)^{1/3}$$

Longitudinal CSR impedance

$$Z_{\text{CSR}}(k) = \frac{1}{c} \int_0^\infty dz w(z) e^{-ikz} = \frac{Z_0}{4\pi} \frac{2}{3^{1/3}} \Gamma \left( \frac{2}{3} \right) e^{i\pi/6} \frac{k^{1/3}}{R^{2/3}}.$$ 

Geometric impedance saturates at large $k$. 
Longitudinal space charge impedance

A beam of radius $a$ propagates inside a pipe of radius $r_w$ with conducting walls, $a \ll r_w$. For $k \ll \gamma / r_w$,

$$Z_{LSC} \approx i \frac{Z_0 c}{4\pi} \frac{k}{\gamma^2} \left(1 + 2 \ln \frac{r_w}{a} \right)$$

For $\gamma/a \gg k \gg \gamma/r_w$,

$$Z_{LSC} \approx i \frac{Z_0 c}{4\pi} \frac{k}{\gamma^2} \left(1 + 2 \ln \frac{\gamma}{ak} \right)$$

In the limit $k \gg \gamma/a$ the concept of impedance breaks down—it is only valid if the induced field does not change much through the beam cross section (Venturini, 2007; Marinelli, Rosenzweig, 2010). Numerical example: for $a = 100 \mu m$, $\gamma = 500$ the condition $k > \gamma/a$ is satisfied for $\lambda \lesssim 1 \mu m$. (From Edwards and Syphers textbook)
MBI in bunch compressors

The theory of MBI in bunch compressors was developed in 2001-2002: Borland (2001), Heifets, Stupakov and Krinsky (2002), Saldin, Schneidmiller and Yurkov (2002), Huang and Kim (2002). The physics is the same as MWI in rings, but the language is different.

Assume an initial modulation of the beam $n(z) = n_0 + n_i \sin k_i z$. In linear regime, after passage through the chicane, $n(z) = n_0 + n_f \sin k_f z$ with $k_f/k_i = C$–the compression factor. The gain, or amplification factor

$$G(k_i) = \frac{n_f}{n_i}$$
“Klystron instability” model

Saldin et al. developed a simple model for the calculation of the gain factor $G$. They assumed the CSR impedance in the magnets and a cold beam.

Take an initial current perturbation $I = I_0 + I_1 \cos k z$ with $I_1 \ll I_0$. After passage through the first magnet the energy modulation in the beam is $\Delta E = eV = eL_b Z_{CSR}(ck)I_1$.

Propagation from 1 to 2 shifts the particles by $\Delta z = (\Delta E/E)R_{56}(1 \rightarrow 2)$, which induces the density perturbation $I_2 = k\Delta z I_0$ (from continuity eq.). Assume $I_2 \gg I_1$ and neglect $I_1$. After passage through 2 the energy modulation is $\Delta E = eV = 2L_b eZ_{CSR}(\omega)I_2$. Propagation from 2 to 3 shifts the particles by $\Delta z = (\Delta E/E)R_{56}(2 \rightarrow 3)$, which induces the density perturbation $I_3 = k\Delta z I_0$. Assume $I_3 \gg I_2$ and neglect $I_2$. The gain factor $G = I_3/I_1$:

$$G = \frac{2\Gamma^2(2/3)}{3^{5/3}} \frac{I_0}{\gamma I_A} \frac{k^{8/3}|R_{56}^{(ch)}|^2 L_d^2}{R^{4/3}}$$

For a cold beam $G$ increases with $k$. 
More sophisticated models of MBI

More sophisticated models involve solution of the Vlasov equation for the evolution of the distribution function through the system. They take into account the energy spread, finite emittance and the compression effect (energy chirp). They predict that $\sigma_E$ and $\epsilon$ tend to suppress $G$ (Landau damping).

Smearing of microbunching occurs if

$$R_{56} \frac{\sigma_E}{E} \gtrsim \lambda$$
More sophisticated models of MBI

Effect of compression, energy spread in finite emittance adds a factor

\[ e^{-C^2k^2(\sigma_E/E)^2R_{56}^2/2} \times e^{-C^2k^2\epsilon/2\beta}(\beta^2R_{51}^2+R_{52}^2) } \]

Example of the gain calculation for LCLS, \( \sigma_E \sim 3 \) keV

Linac wakefield contribution doubles the CSR gain
Using the laser heater to suppress MBI

Increasing the energy spread suppresses the instability

The laser heater was proposed by Saldin, Schneidmiller and Yurkov (2004).
Using the laser heater to suppress MBI

Example: final long. phase space at 14 GeV for initial 8% uv laser intensity modulation at $\lambda=150 \, \mu m$

No Laser-Heater

Matched Laser-Heater

$2 \times 10^{-3}$

$1 \times 10^{-4}$

Courtesy P. Emma
How to quantitatively describe an MBI unstable beam?

Assume that MBI starts from shot noise. How to quantitatively describe the final state of the beam after the instability develops? For example, how to calculate the expected statistical properties of the images of the mictobunched beam from an OTR screen (or any other radiation)?

To understand the problem consider what is involved in the radiation process.

\[
E_n(t) \rightarrow \hat{E}_\omega e^{i\omega t + ikz_n}, \quad \hat{E}_{\text{total}}(\omega) = \sum_{n=1}^{N} \hat{E}_\omega e^{i\omega t + ikz_n}
\]
Beam radiation

Intensity of the radiation $\sim |\hat{E}_{\text{total}}|^2$

$$\left| \sum_{n=1}^{N} \hat{E}_\omega e^{i\omega t+ikz_n} \right|^2 = |\hat{E}_\omega|^2 \sum_{n,j} e^{ik(z_n-z_k)} = |\hat{E}_\omega|^2 \left( N + \sum_{n\neq j} e^{ik(z_n-z_k)} \right)$$

$$\approx N|\hat{E}_\omega|^2 (1 + N\langle e^{ik(z_1-z_2)} \rangle)$$

The first term is the *incoherent* radiation: its intensity is equal to the intensity of one particle $\times N$. The second one is the *coherent* radiation, it can be as large as $N^2$.

The sum $\sum_{n\neq j} e^{ik(z_n-z_k)} \approx 0$ if two conditions are satisfied

- The bunch is much longer than $\lambda$ (then for a typical electron $kz_n \gg \pi$)
- Positions of different particles are not correlated
Correlations of particle positions in the beam

Coherent effects can amplify the radiation (up to a factor of $\sim N$), but they can also suppress it. The beam with distribution on the left graph radiates incoherently, and the beam with the distribution on the right does not radiate at all at the wavelength $\lambda$.

The explanation is on the next slide.
Correlations of particle positions in the beam

The blue particles are distributed randomly but positions of each red one is shifted by $n\lambda/2$ relative to the blue one with $n$ being an odd number.

This is used for a quiet start in FEL simulation codes. The idea of noise suppression in a beam was studied by Gover and Dyunin (2009) and Litivinenko (2009).
Two-particle distribution function

To evaluate $\langle e^{ik(z_1-z_2)} \rangle$ we need to know a two-particle distribution function $f_2(z_1, z_2)$, such that $f_2(z_1, z_2)\,dz_1 \,dz_2$ gives a probability for the first particle to be located in the interval $z_1, z_1 + dz_1$ and at the same time for the second particle to be located in the interval $z_2, z_1 + dz_2$. Then

$$\langle e^{ik(z_1-z_2)} \rangle = \int dz_1 \,dz_2 f_2(z_1, z_2)e^{ik(z_1-z_2)}.$$ 

The one-particle distribution $f_1$ is obtained from $f_2$ by integration

$$f_1(z) = \int f_2(z, z')\,dz'.$$
Correlations of particle positions in the beam

It is a standard practice to introduce a two-particle correlation function $g_2(z_1, z_2)$ defined by

$$f_2(z_1, z_2) = f_1(z_1)f_1(z_2) + g_2(z_1, z_2).$$

If all particles in the distribution are statistically independent of each other, $g_2 = 0$. This is the case of the “shot noise”. Interaction between particles leads to correlations. Assume that $g_2(z_1 - z_2)$ then

$$\langle e^{ik(z_1-z_2)} \rangle = \int dz_1 dz_2 g_2(z_1 - z_2)e^{ik(z_1-z_2)} \approx L \int_{-\infty}^{\infty} g_2(\xi)e^{ik\xi} d\xi = L \hat{g}_2(k)$$

The intensity of coherent radiation measures the Fourier image of the correlation function in the beam.
Correlations of particle positions in the beam

This formalism is straightforwardly generalized to 3D, \( z \rightarrow (x, y, z, x', y', \eta) \) [\( \eta \) is the relative energy deviation].

All these functions, in general, depend on time \( t \). Time evolution of the one-particle distribution function is governed by the Vlasov equation. The correlation function \( g_2 \) satisfies a dynamic equation which can be obtained from BBGKY system of equations.
Dynamic equations for $g_2$ in plasma

From Ichimaru, "Plasma physics"

$$\mathcal{L}(i) = \mathbf{v}_i \cdot \frac{\partial}{\partial \mathbf{r}_i} + \frac{e}{m} \left[ \mathbf{E}_{\text{ext}}(\mathbf{r}_i) + \frac{\mathbf{v}_i}{c} \times \mathbf{B}_{\text{ext}}(\mathbf{r}_i) \right] \frac{\partial}{\partial \mathbf{v}_i}$$

$$\nu(i, j) = \frac{ne^2}{m} \left( \frac{\partial}{\partial \mathbf{r}_i} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} \right) \cdot \frac{\partial}{\partial \mathbf{v}_i}$$

$$\left\{ \frac{\partial}{\partial t} + \mathcal{L}(1) + \mathcal{L}(2) - \int [\nu(1, 3) + \nu(2, 3)] f_1(3; t) d3 \right\} g_2(1, 2; t)$$

$$\quad - \int \nu(1, 3) f_1(1; t) g_2(2, 3; t) d3 - \int \nu(2, 3) f_1(2; t) g_2(3, 1; t) d3$$

$$= \frac{1}{n} [\nu(1, 2) + \nu(2, 1)] f_1(1; t) f_1(2; t)$$

(neglecting higher order terms in density $n$)

This approach in FEL physics is advocated by Shevchenko and Vinokurov (2003).
Correlations of particle positions in the beam

In some problems, assuming small perturbations, one can avoid using $g_2$ by invoking an equivalent technique: work with the variables and the distribution function at the entrance to the system.

$$f_0(z_1, ..., z_N, \eta_1, ..., \eta_N) \quad f_0(z_1, ..., z_N, \hat{\eta}_1, ..., \hat{\eta}_N) \quad f_0(\hat{z}_1, ..., \hat{z}_N, \hat{\eta}_1, ..., \hat{\eta}_N)$$

It is often reasonable to assume that initially there are no correlations in the beam

$$f_N(z_1, \eta_1, z_2, \eta_2, \ldots, z_N, \eta_N) = f_1(z_1, \eta_1)f_1(z_2, \eta_2)\ldots f_1(z_N, \eta_N)$$

A flat beam with a Gaussian energy distribution:

$$f_1(z, \eta) = \frac{1}{\sqrt{2\pi\sigma_\eta L}} e^{-\eta^2/2\sigma_\eta^2}$$

where $L$ is the length of the bunch.
Correlations of particle positions in the beam

It is easy to express the final coordinates $\hat{z}_i$, $\hat{\eta}_i$ in terms of the initial ones $z_i$, $\eta_i$, and the use averaging over the initial distribution function (Stupakov 2010)

$$\langle e^{ik(z_1-z_2)} \rangle = N \int e^{ik[\hat{z}_1(z_1,\eta_1,z_2,\eta_2,...,z_N,\eta_N)-\hat{z}_2(z_1,\eta_1,z_2,\eta_2,...,z_N,\eta_N)]} f_1(z_1,\eta_1)f_1(z_2,\eta_2)...f_1(z_N,\eta_N)\,dz_1\,dz_2\,...\,dz_N\,d\eta_1\,d\eta_2\,...\,d\eta_N$$

This technique can be used for study of noise amplification in HGHG and EEHG seeding (Stupakov, Huang, Ratner 2010) and for noise suppression (Ratner et al., 2011).
We found noise suppression in the system consisting of a drift where particles interact via Coulomb forces followed by a dispersive system

\[ 1 + N \langle e^{ik(z_1-z_2)} \rangle = (1 - \gamma)^2 \]

with

\[ \gamma = n_0 R_{56} \frac{4\pi r_e L_a}{S\gamma} \]
Conclusions

- A well developed standard theory explains the mechanism behind the microbunching instability caused by CSR and SC wakefields in the system. Based on this theory, a laser heater was proposed and is currently employed at LCLS to suppress the instability. However, remaining MBI makes the OTR diagnostics at LCLS useless.

- To properly address the instability starting from random noise, a more sophisticated theoretical technique of correlation functions is required. The traditional method of Vlasov equation is not sufficient.

- Noise suppression in relativistic beams is a new exciting direction in the MBI studies.